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**Measurement of the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow (\pi^0) \ell^+ \nu$  branching fractions, the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \pi^+ \ell^+ \nu$  form-factor shapes, and determination of  $|V_{ub}|$**

BABAR Collaboration ; del Amo Sanchez, P ; Snoek, H L

**Abstract:** We report the results of a study of the exclusive charmless semileptonic decays,  $B^+ \rightarrow (\pi^0) \ell^+ \nu$  and  $B^0 \rightarrow \pi^- \ell^+ \nu$ , undertaken with approximately  $464 \times 10^6$  BB pairs collected at the  $\Upsilon(4S)$  resonance with the BABAR detector. The analysis uses events in which the signal B decays are reconstructed with a loose neutrino reconstruction technique. We obtain partial branching fractions for  $B^+ \rightarrow \pi^+ \ell^+ \nu$  and  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays in three and 12 bins of  $q^2$ , respectively, from which we extract the  $f_+(q^2)$  form-factor shapes and the total branching fractions  $\mathcal{B}(B^+ \rightarrow \pi^+ \ell^+ \nu) = (0.36 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}) \times 10^{-4}$  and  $\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.42 \pm 0.05_{\text{stat}} \pm 0.07_{\text{syst}}) \times 10^{-4}$ . We also measure  $\mathcal{B}(B^+ \rightarrow \pi^+ \ell^+ \nu) = (0.24 \pm 0.08_{\text{stat}} \pm 0.03_{\text{syst}}) \times 10^{-4}$ . We obtain values for the magnitude of the CKM matrix element  $|V_{ub}|$  using three different QCD calculations.

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Other titles: Measurement of the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow (\pi^0) \ell^+ \nu$  branching fractions, the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \pi^+ \ell^+ \nu$  form-factor shapes, and determination of  $|V_{ub}|$

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# Measurement of the $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$ Branching Fractions, the $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \eta \ell^+ \nu$ Form-Factor Shapes, and Determination of $|V_{ub}|$

P. del Amo Sanchez,<sup>1</sup> J. P. Lees,<sup>1</sup> V. Poireau,<sup>1</sup> E. Prencipe,<sup>1</sup> V. Tisserand,<sup>1</sup> J. Garra Tico,<sup>2</sup> E. Grauges,<sup>2</sup> M. Martinelli,<sup>ab,3</sup> D. A. Milanes,<sup>3</sup> A. Palano,<sup>ab,3</sup> M. Pappagallo,<sup>ab,3</sup> G. Eigen,<sup>4</sup> B. Stugu,<sup>4</sup> L. Sun,<sup>4</sup> D. N. Brown,<sup>5</sup> L. T. Kerth,<sup>5</sup> Yu. G. Kolomensky,<sup>5</sup> G. Lynch,<sup>5</sup> I. L. Osipenkov,<sup>5</sup> H. Koch,<sup>6</sup> T. Schroeder,<sup>6</sup> D. J. Asgeirsson,<sup>7</sup> C. Hearty,<sup>7</sup> T. S. Mattison,<sup>7</sup> J. A. McKenna,<sup>7</sup> A. Khan,<sup>8</sup> A. Randle-Conde,<sup>8</sup> V. E. Blinov,<sup>9</sup> A. R. Buzykaev,<sup>9</sup> V. P. Druzhinin,<sup>9</sup> V. B. Golubev,<sup>9</sup> E. A. Kravchenko,<sup>9</sup> A. P. Onuchin,<sup>9</sup> S. I. Serednyakov,<sup>9</sup> Yu. I. Skovpen,<sup>9</sup> E. P. Solodov,<sup>9</sup> K. Yu. Todyshev,<sup>9</sup> A. N. Yushkov,<sup>9</sup> M. Bondioli,<sup>10</sup> S. Curry,<sup>10</sup> D. Kirkby,<sup>10</sup> A. J. Lankford,<sup>10</sup> M. Mandelkern,<sup>10</sup> E. C. Martin,<sup>10</sup> D. P. Stoker,<sup>10</sup> H. Atmacan,<sup>11</sup> J. W. Gary,<sup>11</sup> F. Liu,<sup>11</sup> O. Long,<sup>11</sup> G. M. Vitug,<sup>11</sup> C. Campagnari,<sup>12</sup> T. M. Hong,<sup>12</sup> D. Kovalskyi,<sup>12</sup> J. D. Richman,<sup>12</sup> C. West,<sup>12</sup> A. M. Eisner,<sup>13</sup> C. A. Heusch,<sup>13</sup> J. Kroseberg,<sup>13</sup> W. S. Lockman,<sup>13</sup> A. J. Martinez,<sup>13</sup> T. Schalk,<sup>13</sup> B. A. Schumm,<sup>13</sup> A. Seiden,<sup>13</sup> L. O. Winstrom,<sup>13</sup> C. H. Cheng,<sup>14</sup> D. A. Doll,<sup>14</sup> B. Echenard,<sup>14</sup> D. G. Hitlin,<sup>14</sup> P. Ongmongkolkul,<sup>14</sup> F. C. Porter,<sup>14</sup> A. Y. Rakin,<sup>14</sup> R. Andreassen,<sup>15</sup> M. S. Dubrovin,<sup>15</sup> G. Mancinelli,<sup>15</sup> B. T. Meadows,<sup>15</sup> M. D. Sokoloff,<sup>15</sup> P. C. Bloom,<sup>16</sup> W. T. Ford,<sup>16</sup> A. Gaz,<sup>16</sup> M. Nagel,<sup>16</sup> U. Nauenberg,<sup>16</sup> J. G. Smith,<sup>16</sup> S. R. Wagner,<sup>16</sup> R. Ayad,<sup>17,\*</sup> W. H. Toki,<sup>17</sup> H. Jasper,<sup>18</sup> T. M. Karbach,<sup>18</sup> A. Petzold,<sup>18</sup> B. Spaan,<sup>18</sup> M. J. Kobel,<sup>19</sup> K. R. Schubert,<sup>19</sup> R. Schwierz,<sup>19</sup> D. Bernard,<sup>20</sup> M. Verderi,<sup>20</sup> P. J. Clark,<sup>21</sup> S. Playfer,<sup>21</sup> J. E. Watson,<sup>21</sup> M. Andreotti,<sup>ab,22</sup> D. Bettoni,<sup>a,22</sup> C. Bozzi,<sup>a,22</sup> R. Calabrese,<sup>ab,22</sup> A. Cecchi,<sup>ab,22</sup> G. Cibinetto,<sup>ab,22</sup> E. Fioravanti,<sup>ab,22</sup> P. Franchini,<sup>ab,22</sup> E. Luppi,<sup>ab,22</sup> M. Munerato,<sup>ab,22</sup> M. Negrini,<sup>ab,22</sup> A. Petrella,<sup>ab,22</sup> L. Piemontese,<sup>a,22</sup> R. Baldini-Ferroli,<sup>23</sup> A. Calcaterra,<sup>23</sup> R. de Sangro,<sup>23</sup> G. Finocchiaro,<sup>23</sup> M. Nicolaci,<sup>23</sup> S. Pacetti,<sup>23</sup> P. Patteri,<sup>23</sup> I. M. Peruzzi,<sup>23,†</sup> M. Piccolo,<sup>23</sup> M. Rama,<sup>23</sup> A. Zallo,<sup>23</sup> R. Contri,<sup>ab,24</sup> E. Guido,<sup>ab,24</sup> M. Lo Vetere,<sup>ab,24</sup> M. R. Monge,<sup>ab,24</sup> S. Passaggio,<sup>a,24</sup> C. Patrignani,<sup>ab,24</sup> E. Robutti,<sup>a,24</sup> S. Tosi,<sup>ab,24</sup> B. Bhuyan,<sup>25</sup> V. Prasad,<sup>25</sup> C. L. Lee,<sup>26</sup> M. Morii,<sup>26</sup> A. Adametz,<sup>27</sup> J. Marks,<sup>27</sup> U. Uwer,<sup>27</sup> F. U. Bernlochner,<sup>28</sup> M. Ebert,<sup>28</sup> H. M. Lacker,<sup>28</sup> T. Lueck,<sup>28</sup> A. Volk,<sup>28</sup> P. D. Dauncey,<sup>29</sup> M. Tibbetts,<sup>29</sup> P. K. Behera,<sup>30</sup> U. Mallik,<sup>30</sup> C. Chen,<sup>31</sup> J. Cochran,<sup>31</sup> H. B. Crawley,<sup>31</sup> L. Dong,<sup>31</sup> W. T. Meyer,<sup>31</sup> S. Prell,<sup>31</sup> E. I. Rosenberg,<sup>31</sup> A. E. Rubin,<sup>31</sup> A. V. Gritsan,<sup>32</sup> Z. J. Guo,<sup>32</sup> N. Arnaud,<sup>33</sup> M. Davier,<sup>33</sup> D. Derkach,<sup>33</sup> J. Firmino da Costa,<sup>33</sup> G. Grosdidier,<sup>33</sup> F. Le Diberder,<sup>33</sup> A. M. Lutz,<sup>33</sup> B. Malaescu,<sup>33</sup> A. Perez,<sup>33</sup> P. Roudeau,<sup>33</sup> M. H. Schune,<sup>33</sup> J. Serrano,<sup>33</sup> V. Sordini,<sup>33,‡</sup> A. Stocchi,<sup>33</sup> L. Wang,<sup>33</sup> G. Wormser,<sup>33</sup> D. J. Lange,<sup>34</sup> D. M. Wright,<sup>34</sup> I. Bingham,<sup>35</sup> C. A. Chavez,<sup>35</sup> J. P. Coleman,<sup>35</sup> J. R. Fry,<sup>35</sup> E. Gabathuler,<sup>35</sup> R. Gamet,<sup>35</sup> D. E. Hutchcroft,<sup>35</sup> D. J. Payne,<sup>35</sup> C. Touramanis,<sup>35</sup> A. J. Bevan,<sup>36</sup> F. Di Lodovico,<sup>36</sup> R. Sacco,<sup>36</sup> M. Sigamani,<sup>36</sup> G. Cowan,<sup>37</sup> S. Paramesvaran,<sup>37</sup> A. C. Wren,<sup>37</sup> D. N. Brown,<sup>38</sup> C. L. Davis,<sup>38</sup> A. G. Denig,<sup>39</sup> M. Fritsch,<sup>39</sup> W. Gradl,<sup>39</sup> A. Hafner,<sup>39</sup> K. E. Alwyn,<sup>40</sup> D. Bailey,<sup>40</sup> R. J. Barlow,<sup>40</sup> G. Jackson,<sup>40</sup> G. D. Lafferty,<sup>40</sup> J. Anderson,<sup>41</sup> R. Cenci,<sup>41</sup> A. Jawahery,<sup>41</sup> D. A. Roberts,<sup>41</sup> G. Simi,<sup>41</sup> J. M. Tuggle,<sup>41</sup> C. Dallapiccola,<sup>42</sup> E. Salvati,<sup>42</sup> R. Cowan,<sup>43</sup> D. Dujmic,<sup>43</sup> G. Sciolla,<sup>43</sup> M. Zhao,<sup>43</sup> D. Lindemann,<sup>44</sup> P. M. Patel,<sup>44</sup> S. H. Robertson,<sup>44</sup> M. Schram,<sup>44</sup> P. Biassoni,<sup>ab,45</sup> A. Lazzaro,<sup>ab,45</sup> V. Lombardo,<sup>a,45</sup> F. Palombo,<sup>ab,45</sup> S. Stracka,<sup>ab,45</sup> L. Cremaldi,<sup>46</sup> R. Godang,<sup>46,§</sup> R. Kroeger,<sup>46</sup> P. Sonnek,<sup>46</sup> D. J. Summers,<sup>46</sup> X. Nguyen,<sup>47</sup> M. Simard,<sup>47</sup> P. Taras,<sup>47</sup> B. Viaud,<sup>47</sup> G. De Nardo,<sup>ab,48</sup> D. Monorchio,<sup>ab,48</sup> G. Onorato,<sup>ab,48</sup> C. Sciacca,<sup>ab,48</sup> G. Raven,<sup>49</sup> H. L. Snoek,<sup>49</sup> C. P. Jessop,<sup>50</sup> K. J. Knoepfel,<sup>50</sup> J. M. LoSecco,<sup>50</sup> W. F. Wang,<sup>50</sup> L. A. Corwin,<sup>51</sup> K. Honscheid,<sup>51</sup> R. Kass,<sup>51</sup> J. P. Morris,<sup>51</sup> N. L. Blount,<sup>52</sup> J. Brau,<sup>52</sup> R. Frey,<sup>52</sup> O. Igonkina,<sup>52</sup> J. A. Kolb,<sup>52</sup> R. Rahmat,<sup>52</sup> N. B. Sinev,<sup>52</sup> D. Strom,<sup>52</sup> J. Strube,<sup>52</sup> E. Torrence,<sup>52</sup> G. Castelli,<sup>ab,53</sup> E. Feltres,<sup>ab,53</sup> N. Gagliardi,<sup>ab,53</sup> M. Margoni,<sup>ab,53</sup> M. Morandin,<sup>a,53</sup> M. Posocco,<sup>a,53</sup> M. Rotondo,<sup>a,53</sup> F. Simonetto,<sup>ab,53</sup> R. Stroili,<sup>ab,53</sup> E. Ben-Haim,<sup>54</sup> G. R. Bonneaud,<sup>54</sup> H. Briand,<sup>54</sup> G. Calderini,<sup>54</sup> J. Chauveau,<sup>54</sup> O. Hamon,<sup>54</sup> Ph. Leruste,<sup>54</sup> G. Marchiori,<sup>54</sup> J. Ocariz,<sup>54</sup> J. Prendki,<sup>54</sup> S. Sitt,<sup>54</sup> M. Biasini,<sup>ab,55</sup> E. Manoni,<sup>ab,55</sup> A. Rossi,<sup>ab,55</sup> C. Angelini,<sup>ab,56</sup> G. Batignani,<sup>ab,56</sup> S. Bettarini,<sup>ab,56</sup> M. Carpinelli,<sup>ab,56,¶</sup> G. Casarosa,<sup>ab,56</sup> A. Cervelli,<sup>ab,56</sup> F. Forti,<sup>ab,56</sup> M. A. Giorgi,<sup>ab,56</sup> A. Lusiani,<sup>ac,56</sup> N. Neri,<sup>ab,56</sup> E. Paoloni,<sup>ab,56</sup> G. Rizzo,<sup>ab,56</sup> J. J. Walsh,<sup>a,56</sup> D. Lopes Pegna,<sup>57</sup> C. Lu,<sup>57</sup> J. Olsen,<sup>57</sup> A. J. S. Smith,<sup>57</sup> A. V. Telnov,<sup>57</sup> F. Anulli,<sup>a,58</sup> E. Baracchini,<sup>ab,58</sup> G. Cavoto,<sup>a,58</sup> R. Faccini,<sup>ab,58</sup> F. Ferrarotto,<sup>a,58</sup> F. Ferroni,<sup>ab,58</sup> M. Gaspero,<sup>ab,58</sup> L. Li Gioi,<sup>a,58</sup> M. A. Mazzoni,<sup>a,58</sup> G. Piredda,<sup>a,58</sup> F. Renga,<sup>ab,58</sup> T. Hartmann,<sup>59</sup> T. Leddig,<sup>59</sup> H. Schröder,<sup>59</sup> R. Waldi,<sup>59</sup> T. Adye,<sup>60</sup> B. Franek,<sup>60</sup> E. O. Olaiya,<sup>60</sup> F. F. Wilson,<sup>60</sup> S. Emery,<sup>61</sup> G. Hamel de Monchenault,<sup>61</sup> G. Vasseur,<sup>61</sup> Ch. Yèche,<sup>61</sup> M. Zito,<sup>61</sup> M. T. Allen,<sup>62</sup> D. Aston,<sup>62</sup> D. J. Bard,<sup>62</sup> R. Bartoldus,<sup>62</sup> J. F. Benitez,<sup>62</sup> C. Cartaro,<sup>62</sup> M. R. Convery,<sup>62</sup> J. Dorfan,<sup>62</sup> G. P. Dubois-Felsmann,<sup>62</sup> W. Dunwoodie,<sup>62</sup> R. C. Field,<sup>62</sup> M. Franco Sevilla,<sup>62</sup> B. G. Fulsom,<sup>62</sup> A. M. Gabareen,<sup>62</sup> M. T. Graham,<sup>62</sup> P. Grenier,<sup>62</sup> C. Hast,<sup>62</sup> W. R. Innes,<sup>62</sup> M. H. Kelsey,<sup>62</sup>

H. Kim,<sup>62</sup> P. Kim,<sup>62</sup> M. L. Kocian,<sup>62</sup> D. W. G. S. Leith,<sup>62</sup> S. Li,<sup>62</sup> B. Lindquist,<sup>62</sup> S. Luitz,<sup>62</sup> H. L. Lynch,<sup>62</sup> D. B. MacFarlane,<sup>62</sup> H. Marsiske,<sup>62</sup> D. R. Muller,<sup>62</sup> H. Neal,<sup>62</sup> S. Nelson,<sup>62</sup> C. P. O'Grady,<sup>62</sup> I. Ofte,<sup>62</sup> M. Perl,<sup>62</sup> T. Pulliam,<sup>62</sup> B. N. Ratcliff,<sup>62</sup> A. Roodman,<sup>62</sup> A. A. Salnikov,<sup>62</sup> V. Santoro,<sup>62</sup> R. H. Schindler,<sup>62</sup> J. Schwiening,<sup>62</sup> A. Snyder,<sup>62</sup> D. Su,<sup>62</sup> M. K. Sullivan,<sup>62</sup> S. Sun,<sup>62</sup> K. Suzuki,<sup>62</sup> J. M. Thompson,<sup>62</sup> J. Va'vra,<sup>62</sup> A. P. Wagner,<sup>62</sup> M. Weaver,<sup>62</sup> W. J. Wisniewski,<sup>62</sup> M. Wittgen,<sup>62</sup> D. H. Wright,<sup>62</sup> H. W. Wulsin,<sup>62</sup> A. K. Yarritu,<sup>62</sup> C. C. Young,<sup>62</sup> V. Ziegler,<sup>62</sup> X. R. Chen,<sup>63</sup> W. Park,<sup>63</sup> M. V. Purohit,<sup>63</sup> R. M. White,<sup>63</sup> J. R. Wilson,<sup>63</sup> S. J. Sekula,<sup>64</sup> M. Bellis,<sup>65</sup> P. R. Burchat,<sup>65</sup> A. J. Edwards,<sup>65</sup> T. S. Miyashita,<sup>65</sup> S. Ahmed,<sup>66</sup> M. S. Alam,<sup>66</sup> J. A. Ernst,<sup>66</sup> B. Pan,<sup>66</sup> M. A. Saeed,<sup>66</sup> S. B. Zain,<sup>66</sup> N. Guttman,<sup>67</sup> A. Soffer,<sup>67</sup> P. Lund,<sup>68</sup> S. M. Spanier,<sup>68</sup> R. Eckmann,<sup>69</sup> J. L. Ritchie,<sup>69</sup> A. M. Ruland,<sup>69</sup> C. J. Schilling,<sup>69</sup> R. F. Schwitters,<sup>69</sup> B. C. Wray,<sup>69</sup> J. M. Izen,<sup>70</sup> X. C. Lou,<sup>70</sup> F. Bianchi<sup>ab,71</sup> D. Gamba<sup>ab,71</sup> M. Pelliccioni<sup>ab,71</sup> M. Bomben<sup>ab,72</sup> L. Lanceri<sup>ab,72</sup> L. Vitale<sup>ab,72</sup> N. Lopez-March,<sup>73</sup> F. Martinez-Vidal,<sup>73</sup> A. Oyanguren,<sup>73</sup> J. Albert,<sup>74</sup> Sw. Banerjee,<sup>74</sup> H. H. F. Choi,<sup>74</sup> K. Hamano,<sup>74</sup> G. J. King,<sup>74</sup> R. Kowalewski,<sup>74</sup> M. J. Lewczuk,<sup>74</sup> C. Lindsay,<sup>74</sup> I. M. Nugent,<sup>74</sup> J. M. Roney,<sup>74</sup> R. J. Sobie,<sup>74</sup> T. J. Gershon,<sup>75</sup> P. F. Harrison,<sup>75</sup> T. E. Latham,<sup>75</sup> E. M. T. Puccio,<sup>75</sup> H. R. Band,<sup>76</sup> S. Dasu,<sup>76</sup> K. T. Flood,<sup>76</sup> Y. Pan,<sup>76</sup> R. Prepost,<sup>76</sup> C. O. Vuosalo,<sup>76</sup> and S. L. Wu<sup>76</sup>

(The BABAR Collaboration)

<sup>1</sup>Laboratoire d'Annecy-le-Vieux de Physique des Particules (LAPP),  
Université de Savoie, CNRS/IN2P3, F-74941 Annecy-Le-Vieux, France

<sup>2</sup>Universitat de Barcelona, Facultat de Física, Departament ECM, E-08028 Barcelona, Spain

<sup>3</sup>INFN Sezione di Bari<sup>a</sup>; Dipartimento di Fisica, Università di Bari<sup>b</sup>, I-70126 Bari, Italy

<sup>4</sup>University of Bergen, Institute of Physics, N-5007 Bergen, Norway

<sup>5</sup>Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA

<sup>6</sup>Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany

<sup>7</sup>University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

<sup>8</sup>Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom

<sup>9</sup>Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia

<sup>10</sup>University of California at Irvine, Irvine, California 92697, USA

<sup>11</sup>University of California at Riverside, Riverside, California 92521, USA

<sup>12</sup>University of California at Santa Barbara, Santa Barbara, California 93106, USA

<sup>13</sup>University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA

<sup>14</sup>California Institute of Technology, Pasadena, California 91125, USA

<sup>15</sup>University of Cincinnati, Cincinnati, Ohio 45221, USA

<sup>16</sup>University of Colorado, Boulder, Colorado 80309, USA

<sup>17</sup>Colorado State University, Fort Collins, Colorado 80523, USA

<sup>18</sup>Technische Universität Dortmund, Fakultät Physik, D-44221 Dortmund, Germany

<sup>19</sup>Technische Universität Dresden, Institut für Kern- und Teilchenphysik, D-01062 Dresden, Germany

<sup>20</sup>Laboratoire Leprince-Ringuet, CNRS/IN2P3, Ecole Polytechnique, F-91128 Palaiseau, France

<sup>21</sup>University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom

<sup>22</sup>INFN Sezione di Ferrara<sup>a</sup>; Dipartimento di Fisica, Università di Ferrara<sup>b</sup>, I-44100 Ferrara, Italy

<sup>23</sup>INFN Laboratori Nazionali di Frascati, I-00044 Frascati, Italy

<sup>24</sup>INFN Sezione di Genova<sup>a</sup>; Dipartimento di Fisica, Università di Genova<sup>b</sup>, I-16146 Genova, Italy

<sup>25</sup>Indian Institute of Technology Guwahati, Guwahati, Assam, 781 039, India

<sup>26</sup>Harvard University, Cambridge, Massachusetts 02138, USA

<sup>27</sup>Universität Heidelberg, Physikalisches Institut, Philosophenweg 12, D-69120 Heidelberg, Germany

<sup>28</sup>Humboldt-Universität zu Berlin, Institut für Physik, Newtonstr. 15, D-12489 Berlin, Germany

<sup>29</sup>Imperial College London, London, SW7 2AZ, United Kingdom

<sup>30</sup>University of Iowa, Iowa City, Iowa 52242, USA

<sup>31</sup>Iowa State University, Ames, Iowa 50011-3160, USA

<sup>32</sup>Johns Hopkins University, Baltimore, Maryland 21218, USA

<sup>33</sup>Laboratoire de l'Accélérateur Linéaire, IN2P3/CNRS et Université Paris-Sud 11,  
Centre Scientifique d'Orsay, B. P. 34, F-91898 Orsay Cedex, France

<sup>34</sup>Lawrence Livermore National Laboratory, Livermore, California 94550, USA

<sup>35</sup>University of Liverpool, Liverpool L69 7ZE, United Kingdom

<sup>36</sup>Queen Mary, University of London, London, E1 4NS, United Kingdom

<sup>37</sup>University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom

<sup>38</sup>University of Louisville, Louisville, Kentucky 40292, USA

<sup>39</sup>Johannes Gutenberg-Universität Mainz, Institut für Kernphysik, D-55099 Mainz, Germany

<sup>40</sup>University of Manchester, Manchester M13 9PL, United Kingdom

<sup>41</sup>University of Maryland, College Park, Maryland 20742, USA

<sup>42</sup>University of Massachusetts, Amherst, Massachusetts 01003, USA

<sup>43</sup>Massachusetts Institute of Technology, Laboratory for Nuclear Science, Cambridge, Massachusetts 02139, USA

- <sup>44</sup>McGill University, Montréal, Québec, Canada H3A 2T8
- <sup>45</sup>INFN Sezione di Milano<sup>a</sup>; Dipartimento di Fisica, Università di Milano<sup>b</sup>, I-20133 Milano, Italy
- <sup>46</sup>University of Mississippi, University, Mississippi 38677, USA
- <sup>47</sup>Université de Montréal, Physique des Particules, Montréal, Québec, Canada H3C 3J7
- <sup>48</sup>INFN Sezione di Napoli<sup>a</sup>; Dipartimento di Scienze Fisiche, Università di Napoli Federico II<sup>b</sup>, I-80126 Napoli, Italy
- <sup>49</sup>NIKHEF, National Institute for Nuclear Physics and High Energy Physics, NL-1009 DB Amsterdam, The Netherlands
- <sup>50</sup>University of Notre Dame, Notre Dame, Indiana 46556, USA
- <sup>51</sup>Ohio State University, Columbus, Ohio 43210, USA
- <sup>52</sup>University of Oregon, Eugene, Oregon 97403, USA
- <sup>53</sup>INFN Sezione di Padova<sup>a</sup>; Dipartimento di Fisica, Università di Padova<sup>b</sup>, I-35131 Padova, Italy
- <sup>54</sup>Laboratoire de Physique Nucléaire et de Hautes Energies, IN2P3/CNRS, Université Pierre et Marie Curie-Paris6, Université Denis Diderot-Paris7, F-75252 Paris, France
- <sup>55</sup>INFN Sezione di Perugia<sup>a</sup>; Dipartimento di Fisica, Università di Perugia<sup>b</sup>, I-06100 Perugia, Italy
- <sup>56</sup>INFN Sezione di Pisa<sup>a</sup>; Dipartimento di Fisica, Università di Pisa<sup>b</sup>; Scuola Normale Superiore di Pisa<sup>c</sup>, I-56127 Pisa, Italy
- <sup>57</sup>Princeton University, Princeton, New Jersey 08544, USA
- <sup>58</sup>INFN Sezione di Roma<sup>a</sup>; Dipartimento di Fisica, Università di Roma La Sapienza<sup>b</sup>, I-00185 Roma, Italy
- <sup>59</sup>Universität Rostock, D-18051 Rostock, Germany
- <sup>60</sup>Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom
- <sup>61</sup>CEA, Irfu, SPP, Centre de Saclay, F-91191 Gif-sur-Yvette, France
- <sup>62</sup>SLAC National Accelerator Laboratory, Stanford, California 94309 USA
- <sup>63</sup>University of South Carolina, Columbia, South Carolina 29208, USA
- <sup>64</sup>Southern Methodist University, Dallas, Texas 75275, USA
- <sup>65</sup>Stanford University, Stanford, California 94305-4060, USA
- <sup>66</sup>State University of New York, Albany, New York 12222, USA
- <sup>67</sup>Tel Aviv University, School of Physics and Astronomy, Tel Aviv, 69978, Israel
- <sup>68</sup>University of Tennessee, Knoxville, Tennessee 37996, USA
- <sup>69</sup>University of Texas at Austin, Austin, Texas 78712, USA
- <sup>70</sup>University of Texas at Dallas, Richardson, Texas 75083, USA
- <sup>71</sup>INFN Sezione di Torino<sup>a</sup>; Dipartimento di Fisica Sperimentale, Università di Torino<sup>b</sup>, I-10125 Torino, Italy
- <sup>72</sup>INFN Sezione di Trieste<sup>a</sup>; Dipartimento di Fisica, Università di Trieste<sup>b</sup>, I-34127 Trieste, Italy
- <sup>73</sup>IFIC, Universitat de Valencia-CSIC, E-46071 Valencia, Spain
- <sup>74</sup>University of Victoria, Victoria, British Columbia, Canada V8W 3P6
- <sup>75</sup>Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom
- <sup>76</sup>University of Wisconsin, Madison, Wisconsin 53706, USA

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We report the results of a study of the exclusive charmless semileptonic decays,  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  and  $B^0 \rightarrow \pi^- \ell^+ \nu$ , undertaken with approximately 464 million  $B\bar{B}$  pairs collected at the  $\Upsilon(4S)$  resonance with the *BABAR* detector. The analysis uses events in which the signal  $B$  decays are reconstructed with a loose neutrino reconstruction technique. We obtain partial branching fractions for  $B^+ \rightarrow \eta \ell^+ \nu$  and  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays in three and twelve bins of  $q^2$ , respectively, from which we extract the  $f_+(q^2)$  form-factor shapes and the total branching fractions  $\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu) = (0.36 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}) \times 10^{-4}$  and  $\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.42 \pm 0.05_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-4}$ . We also measure  $\mathcal{B}(B^+ \rightarrow \eta' \ell^+ \nu) = (0.24 \pm 0.08_{\text{stat}} \pm 0.03_{\text{syst}}) \times 10^{-4}$ . We obtain values for the magnitude of the CKM matrix element  $|V_{ub}|$  using three different QCD calculations.

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## I. INTRODUCTION

A precise measurement of the CKM matrix [1] element  $|V_{ub}|$  will constrain the description of weak interactions and CP violation in the Standard Model. The rate for exclusive charmless semileptonic decays involving a scalar meson is proportional to  $|V_{ub} f_+(q^2)|^2$ , where the form factor  $f_+(q^2)$  depends on  $q^2$ , the square of the momentum transferred to the lepton-neutrino pair. Values of  $f_+(q^2)$  are given by unquenched Lattice QCD (LQCD) calculations [2, 3], reliable only at large  $q^2$  ( $\gtrsim 16$  GeV<sup>2</sup>), and by Light Cone Sum Rules (LCSR) calculations [4, 5], based on approximations only valid at small  $q^2$  ( $\lesssim 16$  GeV<sup>2</sup>). The value of  $|V_{ub}|$  can thus be determined by the measurement of partial branching fractions of charmless semileptonic  $B$  decays. Extraction of the  $f_+(q^2)$  form-factor shapes from exclusive decays [6] such as  $B^0 \rightarrow \pi^- \ell^+ \nu$  [7] and  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  may be used to test theoretical calculations [8]. The values of the branching fractions (BF) of the  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  decays will also improve our knowledge of the composition of charmless semileptonic decays and help constrain the size of the gluonic singlet contribution to the form factors for these decays [5].

In this paper, we present measurements of the partial BF's  $\Delta\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu, q^2)$  and  $\Delta\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu, q^2)$  in 3 and 12 bins of  $q^2$ , respectively, as well as the total BF's for all three decay modes. Values of the total BF's were previously reported in Refs. [7, 9–12]. We use the values of  $\Delta\mathcal{B}(q^2)$  for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  mode with form-factor calculations [2–4] to obtain values of  $|V_{ub}|$ . Values of  $|V_{ub}|$  have previously been extracted from  $B^0 \rightarrow \pi^- \ell^+ \nu$  measurements by CLEO [9], *BABAR* [7, 10, 13] and Belle [11]. A very recent measurement by *BABAR* [14] will be discussed in Section VI.

## II. DATA SAMPLE AND SIMULATION

We use a sample of 464 million  $B\bar{B}$  pairs corresponding to an integrated luminosity of 422.6 fb<sup>-1</sup> collected at the  $\Upsilon(4S)$  resonance with the *BABAR* detector [15] at the PEP-II asymmetric-energy  $e^+e^-$  storage rings and a sample of 44 fb<sup>-1</sup> collected approximately 40 MeV below the  $\Upsilon(4S)$  resonance (denoted “off-resonance data”). Detailed Monte Carlo (MC) simulations are used to optimize the signal selections, to estimate the signal efficiencies, and to obtain the shapes of the signal and background distributions. MC samples are generated for  $\Upsilon(4S) \rightarrow B\bar{B}$  events,  $e^+e^- \rightarrow u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}/\tau^+\tau^-$  (continuum) events, and dedicated  $B\bar{B}$  samples containing  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  signal decays. The signal MC events are produced with the FLATQ2 generator [16] and are reweighted to reproduce the  $f_+(q^2, \alpha, c_B)$  Becirevic-Kaidalov (BK) parametrization [17], where the values of the shape and normalization parameters,  $\alpha$  and  $c_B$ , are taken from Ref. [7]. The *BABAR* detector's acceptance and response are simulated using the GEANT4 package [15].

## III. EVENT RECONSTRUCTION AND CANDIDATE SELECTION

We reconstruct the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  decays. The  $\eta$  meson is reconstructed in the  $\eta \rightarrow \gamma\gamma$  and  $\eta \rightarrow \pi^+\pi^-\pi^0$  decay channels (combined BF of 62%) while the  $\eta'$  is reconstructed in the  $\eta' \rightarrow \eta\pi^+\pi^-$  channel, followed by the  $\eta \rightarrow \gamma\gamma$  decay (product BF of 17.5%) [18]. The  $\eta' \rightarrow \rho^0\gamma$  decay channel suffers from large backgrounds and we do not consider it. We carry out an untagged analysis with a loose neutrino reconstruction technique [7], thereby obtaining a large candidate sample.

Event reconstruction with the *BABAR* detector is described in detail elsewhere [15]. Electrons (muons) are identified by their characteristic shower signatures in the electromagnetic calorimeter (muon detector), while charged hadrons are identified using the Cherenkov detector and  $dE/dx$  measurements in the drift chamber. The average electron (muon) reconstruction efficiency is 93% (70%), while its misidentification probability is  $< 0.2\%$  ( $< 1.5\%$ ). The neutrino four-momentum,  $P_\nu =$

\*Now at Temple University, Philadelphia, Pennsylvania 19122, USA

†Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy

‡Also with Università di Roma La Sapienza, I-00185 Roma, Italy

§Now at University of South Alabama, Mobile, Alabama 36688, USA

¶Also with Università di Sassari, Sassari, Italy

$(|\vec{p}_{miss}|, \vec{p}_{miss})$ , is inferred from the difference between the momentum of the colliding-beam particles  $\vec{p}_{beams}$  and the vector sum of the momenta of all the particles detected in a single event  $\vec{p}_{tot}$ , such that  $\vec{p}_{miss} = \vec{p}_{beams} - \vec{p}_{tot}$ . To evaluate  $E_{tot}$ , the energy sum of all the particles, we assume zero mass for all neutrals since photons are difficult to disentangle from neutral hadrons and we take the mass given by the particle identification selectors for the charged particles. In this analysis, we calculate the momentum transfer as  $q^2 = (P_B - P_{meson})^2$  instead of  $q^2 = (P_\ell + P_\nu)^2$ , where  $P_B$ ,  $P_{meson}$  and  $P_\ell$  are the four-momenta of the  $B$  meson, of the  $\pi$ ,  $\eta$  or  $\eta'$  meson, and of the lepton. With this choice, the value of  $q^2$  is unaffected by any mis-reconstruction of the rest of the event. Here  $P_B$  has an effective value. To estimate this value, we first combine the lepton with a  $\pi$ ,  $\eta$  or  $\eta'$  meson to form the so-called  $Y$  pseudo-particle. The angle,  $\theta_{BY}$ , between the  $Y$  and  $B$  momenta in the  $Y(4S)$  frame, can be determined by assuming  $B \rightarrow Y\nu$ . In this frame, the  $Y$  momentum, the  $B$  momentum and the angle  $\theta_{BY}$  define a cone with the  $Y$  momentum as its axis and where the true  $B$  momentum lies somewhere on the surface of the cone. The  $B$  rest frame is thus known up to an azimuthal angle  $\phi$  defined with respect to the  $Y$  momentum. The value of  $q^2$  is then computed as the average of four  $q^2$  values corresponding to four possible angles,  $\phi$ ,  $\phi + \pi/2$ ,  $\phi + \pi$ ,  $\phi + 3\pi/2$  rad, where the angle  $\phi$  is chosen randomly [19]. We note that,  $\theta_{BY}$  being a real angle,  $|\cos \theta_{BY}| \leq 1$ . We correct for the reconstruction effects on the  $q^2$  resolution ( $0.51 \text{ GeV}^2$ ) by applying an unregularized unfolding algorithm to the measured  $q^2$  spectra [20].

The candidate selections are optimized to maximize the ratio  $S/\sqrt{(S+B)}$  in the MC simulation, where  $S$  is the number of signal events and  $B$  is the total number of background events. Continuum background is suppressed by requiring the ratio of second to zeroth Fox-Wolfram moments [21] to be smaller than 0.5. This background is further suppressed for  $B^0 \rightarrow \pi^-\ell^+\nu$  by selections on the number of charged particle tracks and neutral calorimeter clusters [22] that reject radiative Bhabha and converted photon processes. We ensure that the momenta of the lepton and meson candidates are kinematically compatible with a real signal decay by requiring that a geometrical vertex fit of the two particles gives a  $\chi^2$  probability greater than 0.01 and that their angles in the laboratory frame be between 0.41 and 2.46 rad with respect to the  $e^-$ -beam direction, the acceptance of the detector. To avoid  $J/\psi \rightarrow \mu^+\mu^-$  decays, we reject  $B^0 \rightarrow \pi^-\mu^+\nu$  candidates if the  $Y$  mass corresponds to the  $J/\psi$  mass. The electron (muon) tracks are required to have momenta greater than 0.5 (1.0) GeV in the laboratory frame to reduce misidentified leptons and secondary decays such as  $D \rightarrow X\ell\nu$ ,  $J/\psi$ ,  $\tau$  and kaon decays. Furthermore, the momenta of the lepton and the meson are restricted to enhance signal over background. We require: for  $B^0 \rightarrow \pi^-\ell^+\nu$  decays,  $|\vec{p}_\ell^*| > 2.2 \text{ GeV}$  or  $|\vec{p}_\pi^*| > 1.3 \text{ GeV}$  or  $|\vec{p}_\ell^*| + |\vec{p}_\pi^*| > 2.8 \text{ GeV}$ ; for  $B^+ \rightarrow \eta\ell^+\nu$

decays,  $|\vec{p}_\ell^*| > 2.1 \text{ GeV}$  or  $|\vec{p}_\eta^*| > 1.3 \text{ GeV}$  or  $|\vec{p}_\ell^*| + |\vec{p}_\eta^*| > 2.8 \text{ GeV}$ ; and for  $B^+ \rightarrow \eta'\ell^+\nu$  decays,  $|\vec{p}_\ell^*| > 2.0 \text{ GeV}$  or  $|\vec{p}_{\eta'}^*| > 1.65 \text{ GeV}$  or  $0.69 \times |\vec{p}_\ell^*| + |\vec{p}_{\eta'}^*| > 2.4 \text{ GeV}$  (all asterisked variables are in the CM). For the  $B^+ \rightarrow \eta^{(\prime)}\ell^+\nu$  decays, we restrict the reconstructed masses of the  $\eta'$  and  $\eta$  to lie in the intervals  $0.92 < m_{\eta'} < 0.98 \text{ GeV}$  and  $0.51 < m_\eta < 0.57 \text{ GeV}$ . For these decays, we also reject events with  $q^2$  higher than  $16 \text{ GeV}^2$ , since the signal is dominated by background in that range. Most backgrounds are reduced by  $q^2$ -dependent selections on the angle between the thrust axes of the  $Y$  and of the rest of the event, on the polar angle associated with  $\vec{p}_{miss}$ , on the invariant missing mass squared ( $m_{miss}^2 = E_{miss}^2 - |\vec{p}_{miss}|^2$ ) divided by twice the missing energy ( $E_{miss} = E_{beams} - E_{tot}$ ), and on the helicity angle ( $\cos \theta_\ell$ ), the angle between the direction of the  $W$  boson ( $\ell$  and  $\nu$  combined) in the rest frame of the  $B$  meson, and the direction of the lepton in the rest frame of the  $W$  boson. For  $B^+ \rightarrow \eta\ell^+\nu$  decays, more background is rejected by requiring that  $|\cos \theta_V| < 0.95$ , where  $\theta_V$  is the helicity angle of the  $\eta$  meson [16].

The kinematic variables  $\Delta E = (P_B \cdot P_{beams} - s/2)/\sqrt{s}$  and  $m_{ES} = \sqrt{(s/2 + \vec{p}_B \cdot \vec{p}_{beams})^2/E_{beams}^2 - \vec{p}_B^2}$  are used in a two-dimensional extended maximum-likelihood fit [23] to separate signal from background. Here,  $\sqrt{s}$  is the center-of-mass energy of the colliding particles and  $P_B = P_{meson} + P_\ell + P_\nu$ , in the laboratory frame. We only retain candidates with  $|\Delta E| < 1.0 \text{ GeV}$  and  $m_{ES} > 5.19 \text{ GeV}$ , thereby removing the region with large backgrounds from the fit. On average, less than 1.14 candidates is observed per event. For events with multiple candidates, only the candidate with the largest value of  $\cos \theta_\ell$  is kept. The signal event reconstruction efficiency varies between 8.3% and 14.6% for  $B^0 \rightarrow \pi^-\ell^+\nu$ , and 1.4% and 2.6% for  $B^+ \rightarrow \eta\ell^+\nu$  decays ( $\gamma\gamma$  channel), depending on  $q^2$ . It is 0.6% for both  $B^+ \rightarrow \eta\ell^+\nu$  ( $\pi^+\pi^-\pi^0$  channel) and  $B^+ \rightarrow \eta'\ell^+\nu$  decays.

#### IV. BACKGROUNDS AND SIGNAL EXTRACTION

Backgrounds can be broadly grouped into three main categories: decays arising from  $b \rightarrow u\ell\nu$  transitions (other than the signal), decays in other  $B\bar{B}$  events (excluding  $b \rightarrow u\ell\nu$ ) and decays in continuum events. For the  $B^0 \rightarrow \pi^-\ell^+\nu$  mode only, in which there are many events, each of the first two categories is further split into a background category where the pion and the lepton come from the decay of the same  $B$ , and a background category where the pion and the lepton come from the decay of different  $B$  mesons.

Given the sufficient number of events for the  $\pi\ell\nu$  decay mode, the data samples can be subdivided in 12 bins of  $q^2$  for the signal and 2 bins for each of the five background categories. Two bins are used for each background category since the background  $q^2$  spectra are not that well known and need to be adjusted in the fit when the num-

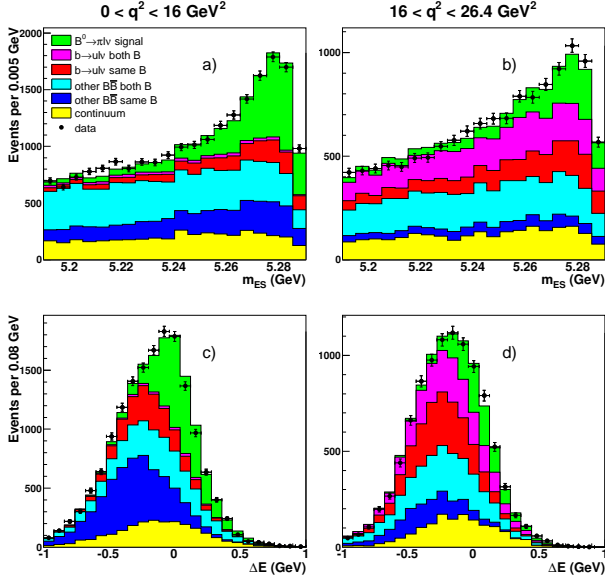


FIG. 1: (color online) Projections of the data and fit results for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays, in the signal-enhanced region: (a,b)  $m_{ES}$  with  $-0.16 < \Delta E < 0.20$  GeV; and (c,d)  $\Delta E$  with  $m_{ES} > 5.268$  GeV. The distributions (a,c) and (b,d) are projections for  $q^2 < 16$  GeV<sup>2</sup> and for  $q^2 > 16$  GeV<sup>2</sup>, respectively

TABLE I: Fitted yields in the full  $q^2$  range for the signal and each background category, total number of MC and data events, and values of  $\chi^2$  for the fit region.

Decay mode	$\pi^- \ell^+ \nu$	$\eta \ell^+ \nu$	$\eta' \ell^+ \nu$
Signal	$11778 \pm 435$	$888 \pm 98$	$141 \pm 46$
$b \rightarrow u \ell \nu$	$27793 \pm 929$	$2201(fixed)$	$204(fixed)$
Other $B\bar{B}$	$80185 \pm 963$	$17429 \pm 247$	$2660 \pm 82$
Continuum	$27790 \pm 814$	$3435 \pm 195$	$517(fixed)$
MC events	$147546 \pm 467$	$23953 \pm 183$	$3522 \pm 68$
Data events	$147529 \pm 384$	$23952 \pm 155$	$3517 \pm 59$
$\chi^2/ndf$	$411/386$	$56/52$	$19/17$

ber of events is sufficiently large to permit it. The  $q^2$  range of the background binning for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decay is  $[0, 18, 26.4]$  GeV<sup>2</sup> for the  $b \rightarrow u \ell \nu$  same  $B$  category,  $[0, 22, 26.4]$  GeV<sup>2</sup> for the  $b \rightarrow u \ell \nu$  both  $B$  category,  $[0, 10, 26.4]$  GeV<sup>2</sup> for the other  $B\bar{B}$  same  $B$  category,  $[0, 14, 26.4]$  GeV<sup>2</sup> for the other  $B\bar{B}$  both  $B$  category and  $[0, 22, 26.4]$  GeV<sup>2</sup> for the continuum category. In each case, the  $q^2$  ranges of the two bins are chosen to contain a similar number of events. For the  $\eta^{(\prime)} \ell \nu$  modes, a smaller number of events leads us to restrict the signal and each of the three background categories to a single  $q^2$  bin except for the signal in the  $\eta \ell \nu$  mode where  $\eta \rightarrow \gamma \gamma$ , which is investigated in 3 bins of  $q^2$ .

We use the  $\Delta E$ - $m_{ES}$  histograms, obtained from the MC simulation as two-dimensional probability density functions (PDFs), in our fit to the data, to extract the

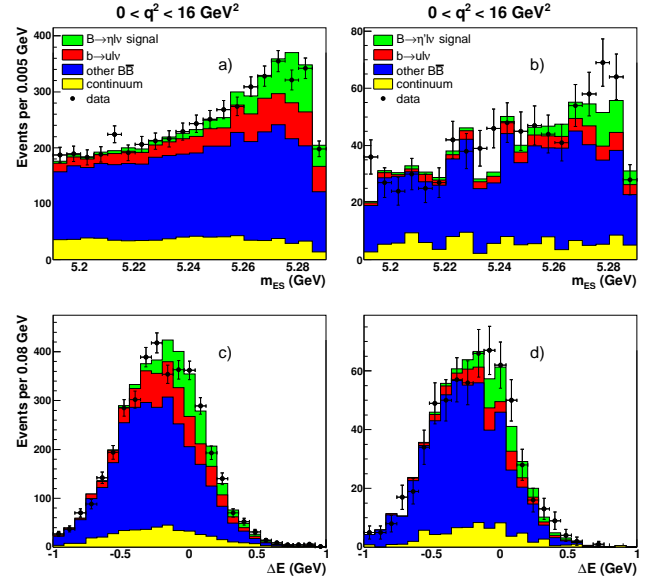


FIG. 2: (color online) Projections of the data and fit results for the  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  decays, in the signal-enhanced region: (a,b)  $m_{ES}$  with  $-0.16 < \Delta E < 0.20$  GeV; and (c,d)  $\Delta E$  with  $m_{ES} > 5.268$  GeV. The distributions (a,c) and (b,d) are projections for the  $B^+ \rightarrow \eta \ell^+ \nu$  and  $B^+ \rightarrow \eta' \ell^+ \nu$  decays, respectively, both for  $q^2 < 16$  GeV<sup>2</sup>.

yields of the signal and backgrounds as a function of  $q^2$ . The MC continuum background yield and shape are first corrected to match the yield and shape of the off-resonance data control sample. Because of an insufficient number of events, the  $b \rightarrow u \ell \nu$  background is fixed in the fit for the  $\eta^{(\prime)} \ell \nu$  modes, and the continuum contribution is also fixed for the  $\eta \ell \nu(3\pi)$  and  $\eta' \ell \nu$  modes. Whenever a background is not varied in the fit, it is fixed to the MC prediction except for the continuum background which is fixed to its corrected yield and shape. The background parameters which are free in the fit require an adjustment of less than 10% with respect to the MC predictions. For illustration purposes only, we show in Fig. 1  $\Delta E$  and  $m_{ES}$  fit projections in the signal-enhanced region for  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays in two ranges of  $q^2$  corresponding to the sum of eight bins below and four bins above  $q^2 = 16$  GeV<sup>2</sup>, respectively. More detailed  $\Delta E$  and  $m_{ES}$  fit projections in each  $q^2$  bin are also shown in Figs. 5 and 6 of the Appendix for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays. The data and the fit results are in good agreement. Fit projections for  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$ , only available below  $q^2 = 16$  GeV<sup>2</sup>, are shown in Fig. 2. Table I shows the total fitted yields in the full  $q^2$  range for the signal and each background category as well as the  $\chi^2$  values and degrees of freedom for the overall fit region. The yield values in the  $B^+ \rightarrow \eta \ell^+ \nu$  column are the result of the fit to the combined  $\gamma \gamma$  and  $3\pi$  modes. The fixed values are given by the MC simulation.

TABLE II: Values of signal yields,  $\Delta\mathcal{B}(q^2)$  and their relative uncertainties (%) for  $B^0 \rightarrow \pi^- \ell^+ \nu$ ,  $B^+ \rightarrow \eta \ell^+ \nu$  and  $B^+ \rightarrow \eta' \ell^+ \nu$  decays.

Decay mode	$\pi^- \ell^+ \nu$				$\eta \ell^+ \nu$	$\eta' \ell^+ \nu$
$q^2$ range (GeV <sup>2</sup> )	$q^2 < 12$	$q^2 < 16$	$q^2 > 16$	full $q^2$ range	$q^2 < 16$	$q^2 < 16$
Yield	6541.6	8422.1	3355.4	11777.6	887.9	141.0
BF ( $10^{-4}$ )	0.83	1.09	0.33	1.42	0.36	0.24
Fit error	3.9	3.7	7.6	3.5	12.5	32.8
Detector effects	3.1	3.5	6.1	4.0	8.0	8.8
Continuum bkg	2.3	1.9	4.0	2.4	0.3	7.1
$B \rightarrow X_u \ell \nu$ bkg	2.0	1.7	4.2	2.0	7.6	6.7
$B \rightarrow X_c \ell \nu$ bkg	0.6	0.7	1.8	1.0	1.2	2.6
Other effects	2.3	2.2	3.2	2.3	3.4	4.6
Total uncertainty	6.3	6.2	12.0	6.7	17.0	35.8

## V. SYSTEMATIC UNCERTAINTIES

Systematic uncertainties on the values of the partial branching fractions,  $\Delta\mathcal{B}(q^2)$ , and their correlations among the  $q^2$  bins have been investigated. These uncertainties are estimated from the variations of the resulting partial BF values (or total BF values for  $B^+ \rightarrow \eta' \ell^+ \nu$  decays) when the data are re-analyzed with different simulation parameters and reweightings. For each parameter, we use the full MC dataset to generate new  $\Delta E$ - $m_{ES}$  distributions (“MC event samples”) by varying randomly only the parameter of interest over a complete ( $> 3\sigma$ ) gaussian distribution whose standard deviation is given by the uncertainty on the specific parameter under investigation. One hundred such samples are generated for each parameter. Uncertainties due to  $B$  counting and final state radiation are estimated by generating only one sample. Each MC sample is analyzed the same way as real data to determine values of  $\Delta\mathcal{B}(q^2)$  (or total BF values for  $B^+ \rightarrow \eta' \ell^+ \nu$  decays). The contribution of the parameter to the systematic uncertainty is given by the RMS value of the distribution of these values over the one hundred samples.

The systematic uncertainties due to the imperfect description of the detector in the simulation are computed by using the uncertainties, determined from control samples, on the tracking efficiency of all charged particle tracks, on the particle identification efficiencies of signal candidate tracks, on the calorimeter efficiencies (varied separately for photons and  $K_L^0$ ), on the energy deposited in the calorimeter by  $K_L^0$  mesons as well as on their production spectrum. The reconstruction of these neutral particles affects the analysis through the neutrino reconstruction. The uncertainties due to the generator-level inputs to the simulation are given by the uncertainties in the BFs of the background processes  $b \rightarrow u \ell \nu$  and  $b \rightarrow c \ell \nu$ , in the BFs of the secondary decays producing leptons [18], and in the BFs of the  $\Upsilon(4S) \rightarrow B\bar{B}$  decays [8]. The  $B \rightarrow X \ell \nu$  form factor uncertainties, where  $X = (\pi, \rho, \omega, \eta^{(\prime)}, D, D^*)$ , are given by recent calculations or measurements [18]. The uncertainties in the heavy

quark parameters used in the simulation of non-resonant  $b \rightarrow u \ell \nu$  events are given in Ref. [24]. We assign an uncertainty of 20% [25] to the final state radiation (FSR) corrections calculated by PHOTOS [26]. Finally, the uncertainties due to the modeling of the continuum are established by using the uncertainties in its  $q^2$ ,  $\Delta E$  and  $m_{ES}$  shapes and total yield given by comparisons with the off-resonance data control sample.

The list of all the systematic uncertainties, as well as their values for the partial and total BFs, are given in Tables IV and V of the Appendix. The item “Signal MC stat error” in these tables includes the systematic uncertainty due to the unfolding procedure. The correlation matrices obtained in the measurement of the partial BFs are presented in Tables VI, VII and VIII. A condensed version of all the uncertainties is given in Table II together with signal yields and partial BFs in selected  $q^2$  ranges. The values given for the  $B^+ \rightarrow \eta \ell^+ \nu$  decays are those obtained from the fits to the distributions of the  $\eta \rightarrow \gamma\gamma$  and  $\eta \rightarrow \pi^+ \pi^- \pi^0$  channels combined. The larger relative uncertainties occurring in bin 12 of Table IV are due to poorly reconstructed events, and to the small raw yield in that bin. The former arises from the presence of a large number of low momentum pions and a large background. This makes it difficult to select the right pion and results in a larger absolute uncertainty on the fitted yield. The small yield leads to a fairly large unfolding correction in this bin and thus to a considerably reduced unfolded yield. On the other hand, the unfolding process increases the absolute uncertainty only slightly. The reduced yield together with the larger absolute uncertainty lead to the larger relative uncertainties reported in the table.

## VI. RESULTS

The partial BFs are calculated for  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \eta \ell^+ \nu$  decays using the unfolded signal yields, the signal efficiencies given by the simulation and the BFs  $\mathcal{B}(\Upsilon(4S) \rightarrow B^0 \bar{B}^0) = 0.484 \pm 0.006$  and  $\mathcal{B}(\Upsilon(4S) \rightarrow$



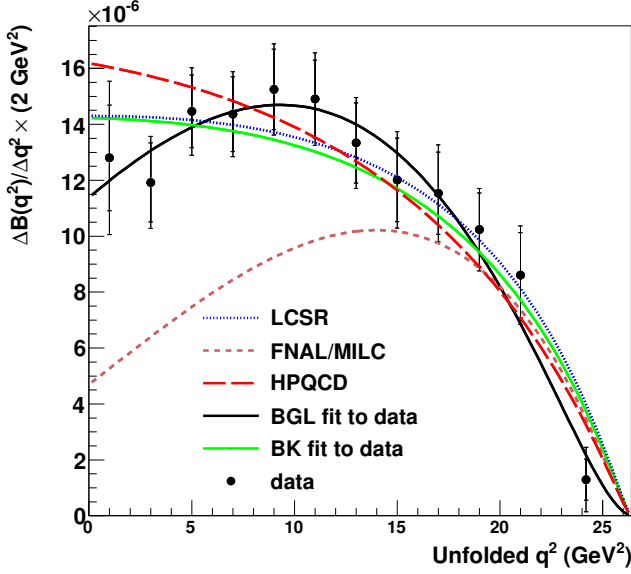


FIG. 3: (color online) Partial  $\Delta\mathcal{B}(q^2)$  spectrum in 12 bins of  $q^2$  for  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays. The data points are placed in the middle of each bin whose width is defined in Table IV. The smaller error bars are statistical only while the larger ones also include systematic uncertainties. The solid green and black curves show the result of the fit to the data of the BK [17] and BGL [27] parametrizations, respectively. The data are also compared to unquenched LQCD calculations (HPQCD [2], FNAL [3]) and an LCSR calculation [4].

$B^+ B^-$ ) =  $0.516 \pm 0.006$  [8]. We obtain the total BF's  $\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.42 \pm 0.05_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-4}$ ,  $\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu) = (0.36 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}) \times 10^{-4}$  and  $\mathcal{B}(B^+ \rightarrow \eta' \ell^+ \nu) = (0.24 \pm 0.08_{\text{stat}} \pm 0.03_{\text{syst}}) \times 10^{-4}$ . The BF value for  $B^+ \rightarrow \eta' \ell^+ \nu$  has a significance of  $2.7\sigma$ . The BF value, obtained from a fit to the combined  $\gamma\gamma$  and  $3\pi$  channels of the  $B^+ \rightarrow \eta \ell^+ \nu$  decays, is in excellent agreement with the weighted average of the total BF's obtained separately for the  $\gamma\gamma$  and  $3\pi$  channels. Consistent results are obtained when dividing the final data set into chronologically-ordered subsets, electron only and muon only subsets, modifying the  $q^2$  or the  $\Delta E$  and  $m_{\text{ES}}$  binings, and varying the event selection requirements.

The experimental  $\Delta\mathcal{B}(q^2)$  distributions are displayed in Fig. 3 for  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays and in Fig. 4 for  $B^+ \rightarrow \eta \ell^+ \nu$  decays, together with theoretical predictions. These distributions have been modified to remove FSR effects to allow a direct comparison with the theoretical predictions which do not include such effects. We obtain the  $f_+(q^2)$  shape from a fit to these distributions. The  $\chi^2$  function minimized in the fit to the  $f_+(q^2)$  shape uses the BGL parametrization [27] consisting of a two-parameter polynomial expansion. For the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays, the fit gives  $a_1/a_0 = -0.64 \pm 0.30$  and  $a_2/a_0 = -6.8 \pm 1.8$ , with  $P(\chi^2) = 92.2\%$  as well as a value of  $|V_{ub} f_+(0)| = (8.6 \pm 0.3_{\text{stat}} \pm 0.3_{\text{syst}}) \times 10^{-4}$  from the fit extrapolated to  $q^2 = 0$ . This value can be used to predict

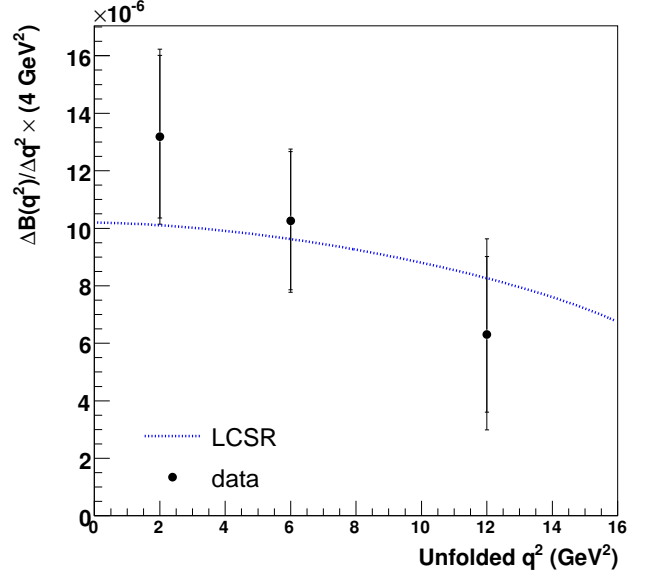


FIG. 4: (color online) Partial  $\Delta\mathcal{B}(q^2)$  spectrum in 3 bins of  $q^2$  for  $B^+ \rightarrow \eta \ell^+ \nu$  decays. The data points are placed in the middle of each bin whose width is defined in Table V. The smaller error bars are statistical only while the larger ones also include systematic uncertainties. The data are also compared to an LCSR calculation [5].

rates of other decays such as  $B \rightarrow \pi\pi$  [28]. For completeness, we also show the fit to the BK parametrization [17], which gives  $\alpha_{BK} = 0.51 \pm 0.04$ , with  $P(\chi^2) = 39.6\%$ .

Since we are only interested in the shape of the form factors, we first normalize the form factor predictions to the experimental data by requiring the integrals of both to be the same over the  $q^2$  ranges of validity given in Table III for each theory. Considering only experimental uncertainties, we then calculate the  $\chi^2$  probabilities relative to the binned data result for various theoretical predictions. The comparison between theory and experiment is restricted to the  $q^2$  ranges of validity. We obtain  $P(\chi^2) = 30.1\%$  for HPQCD [2],  $18.4\%$  for FNAL [3] and  $47.0\%$  for LCSR [4] for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays. As shown in Fig. 4, an LCSR calculation [5] is compatible with the data for the  $B^+ \rightarrow \eta \ell^+ \nu$  decays. It should be noted that the theoretical curves in Fig. 3 have been extrapolated over the full  $q^2$  range based on a parametrization obtained over their  $q^2$  ranges of validity. These extended ranges are only meant to illustrate a possible extension of the present theoretical calculations.

We extract a value of  $|V_{ub}|$  from the  $B^0 \rightarrow \pi^- \ell^+ \nu$   $\Delta\mathcal{B}(q^2)$  distributions using the relation:  $|V_{ub}| = \sqrt{\Delta\mathcal{B}/(\tau_{B^0} \Delta\zeta)}$ , where  $\Delta\mathcal{B}$  and  $\Delta\zeta$  are restricted to the  $q^2$  ranges of validity given in Table III,  $\tau_{B^0} = 1.525 \pm 0.009$  ps [8] is the  $B^0$  lifetime and  $\Delta\zeta = \Gamma/|V_{ub}|^2$  is the normalized partial decay rate predicted by the form-factor calculations [2–4]. These values of  $\Delta\zeta$  are independent of experimental data. The values of  $|V_{ub}|$  given

TABLE III: Values of  $|V_{ub}|$  derived from the form-factor calculations for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays. The three uncertainties on  $|V_{ub}|$  are statistical, systematic and theoretical, respectively.

	$q^2$ (GeV <sup>2</sup> )	$\Delta\mathcal{B}$ (10 <sup>-4</sup> )	$\Delta\zeta$ (ps <sup>-1</sup> )	$ V_{ub} $ (10 <sup>-3</sup> )	$\chi^2/\text{ndf}$	$\text{Prob}(\chi^2)$
HPQCD [2]	> 16	$0.33 \pm 0.03 \pm 0.03$	$2.07 \pm 0.57$	$3.24 \pm 0.13 \pm 0.16^{+0.57}_{-0.37}$	4.9/4	30.1%
FNAL [3]	> 16	$0.33 \pm 0.03 \pm 0.03$	$2.21^{+0.47}_{-0.42}$	$3.14 \pm 0.12 \pm 0.16^{+0.35}_{-0.29}$	6.2/4	18.4%
LCSR [4]	< 12	$0.84 \pm 0.03 \pm 0.04$	$4.00^{+1.01}_{-0.95}$	$3.70 \pm 0.07 \pm 0.09^{+0.54}_{-0.39}$	5.6/6	47.0%

in Table III range from  $(3.1 - 3.7) \times 10^{-3}$ . A value of  $|V_{ub}|$  could not be obtained from the  $B^+ \rightarrow \eta \ell^+ \nu$  decays because the required theoretical input,  $\Delta\zeta$ , is not yet available.

It is of interest to note that, at first glance, there appears to be a large overlap between the present analysis of the  $B^0 \rightarrow \pi^- \ell^+ \nu$  data and that of another recent *BABAR* measurement [14]. However, there are significant differences between the two analyses. The overlap of the selected event samples is estimated to be less than 20%, the event reconstruction and simulation are somewhat different, and almost all of the systematic uncertainties are expected to be highly correlated. Combining the results of the two analyses is thus a complex process requiring further study. Nevertheless, it is gratifying to note that the total BF as well as the values of  $|V_{ub}|$  obtained in the two analyses are consistent with each other. For example, the present BF is  $\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.42 \pm 0.05_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-4}$  while the corresponding BF for the other analysis is  $(1.44 \pm 0.06 \pm 0.07) \times 10^{-4}$  and it is  $(1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$  when combining the charged and neutral pion channels, assuming isospin symmetry.

## VII. SUMMARY

In summary, we have measured the partial BFs of  $B^+ \rightarrow \eta \ell^+ \nu$  in 3 bins of  $q^2$  and of  $B^0 \rightarrow \pi^- \ell^+ \nu$  in 12 bins of  $q^2$ . From these distributions, we extract the  $f_+(q^2)$  shapes which are found to be compatible with all three theoretical predictions for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays and with the LCSR calculation for the  $B^+ \rightarrow \eta \ell^+ \nu$  decays. The BGL parametrization fits our data well and allows us to obtain the value of  $|V_{ub} f_+(0)|$ . Our measured branching fractions of the three decays under study lead to a significant improvement in our knowledge of the composition of the inclusive charmless semileptonic decay rate. Our value of the total BF for  $B^+ \rightarrow \eta \ell^+ \nu$ , with a significance of  $2.7\sigma$ , is an order of magnitude smaller than the most recent CLEO result [9]. Our value of the total BF for  $B^+ \rightarrow \eta \ell^+ \nu$  is consistent with a previous un-

tagged *BABAR* result [12]. The value of the ratio  $\mathcal{B}(B^+ \rightarrow \eta' \ell^+ \nu)/\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu) = 0.67 \pm 0.24_{\text{stat}} \pm 0.11_{\text{syst}}$  allows an important gluonic singlet contribution to the  $\eta'$  form factor. The present value of the total BF for  $B^0 \rightarrow \pi^- \ell^+ \nu$  is in very good agreement with a previous untagged *BABAR* measurement [7] as well as with a recent *BABAR* result [14]. It has comparable precision to the present world average [8]. For  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays, we obtain values of  $|V_{ub}|$  for three different QCD calculations. The results are consistent with those of Refs. [7, 14]. The three values are all acceptable according to the data and are consistent, within large theoretical uncertainties, with the value measured in inclusive semileptonic decays:  $|V_{ub}| = (4.27 \pm 0.38) \times 10^{-3}$  [8].

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## VIII. APPENDIX

The list of all the systematic uncertainties, as well as their values for the partial and total BF's, are given in

Tables IV and V for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  decays, respectively. In Table IV, we have one column for each bin of  $q^2$ , three columns for various ranges of  $q^2$  as well as the last column for the global result. In row 1, “Fitted yield”, we give the raw fitted yield as number of events. In row 2, “Yield systematic error”, we give the systematic uncertainty in % for each fitted yield. In row 3, “Yield fit error”, we give the statistical uncertainty in % for each fitted yield. In row 4, “Efficiency”, we give the efficiency in % attached to each yield. In row 5, “Eff. (Without FSR)”, we give the efficiency in %, modified to remove the FSR effect. In row 6, “Unfolded yield”, we give the yields from row 1 unfolded to give the true values of the yields in each bin, expressed as number of events. In row 7, “ $\Delta\mathcal{B}$ ”, we give the values of the partial BF's computed as usual using the true (unfolded) yields and the efficiencies with FSR. In row 8, “ $\Delta\mathcal{B}$  (Without FSR)”, we give the values of the partial BF's computed as usual using the true (unfolded) yields and the efficiencies modified to remove the FSR effect. In rows 9 - 42, we give the contributions in % to the relative systematic uncertainties for each value of  $\Delta\mathcal{B}$  as a function of  $q^2$ . In row 43, “Signal MC statistical error”, we give the statistical uncertainty due to the number of MC signal events. In row 44, “Total systematic error”, we give the total systematic uncertainty in % for each value of  $\Delta\mathcal{B}$ , obtained as the sum in quadrature of all the systematic uncertainties in each column. In row 45, “Fit error”, we give the statistical uncertainty in % for each value of  $\Delta\mathcal{B}$  obtained from propagating the uncertainties on the raw fitted yields, on the unfolding process and on the efficiencies. In row 46, “Total error”, we first give the total uncertainty in % for each value of  $\Delta\mathcal{B}$ , obtained as the sum in quadrature of the total systematic error and the fit error. We then give, in the last four columns, the total uncertainties in % for each range of  $q^2$ , obtained as the sum in quadrature of the total errors for the appropriate number of  $q^2$  bins. A similar description applies to Table V.

In our analysis, we compute the covariance matrix for each source of uncertainty, and use these matrices to calculate the uncertainties on the total BF's. The correlation matrices for the total statistical and systematic uncertainties are given in Table VI for the  $B^+ \rightarrow \eta \ell^+ \nu$  yields and in Tables VII and VIII for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  yields, respectively. Finally, detailed  $\Delta E$  and  $m_{ES}$  fit projections in each  $q^2$  bin are also shown in Figs. 5 and 6, respectively, for the  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays.

TABLE IV:  $B^0 \rightarrow \pi^- \ell^+ \nu$  yields, efficiencies(%),  $\Delta\mathcal{B}$  ( $10^{-7}$ ) and their relative uncertainties (%). The  $\Delta\mathcal{B}$  and efficiency values labelled “Without FSR” are modified to remove FSR effects. This procedure has no significant impact on the  $\Delta\mathcal{B}$  values.

$q^2$ bins (GeV <sup>2</sup> )	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-26.4	$q^2 < 12$	$q^2 < 16$	$q^2 > 16$	Total
Fitted yield	894.7	987.8	1177.1	1181.3	1178.6	1122.1	996.1	884.5	904.3	847.5	729.9	873.9	6541.6	8422.1	3355.4	11777.6
Yield systematic error	13.7	5.9	3.1	2.7	2.8	2.7	2.9	3.6	4.7	5.3	7.9	25.6	4.3	3.0	8.9	4.3
Yield fit error	12.8	8.1	6.0	6.4	6.7	7.0	8.2	9.8	10.3	10.5	14.0	21.0	3.2	3.6	7.9	3.7
Efficiency	8.34	9.10	9.22	9.09	8.59	8.46	8.53	8.50	9.40	10.52	11.61	14.59	-	-	-	-
Eff. (Without FSR)	8.00	8.97	9.15	9.18	8.63	8.53	8.58	8.61	9.45	10.66	11.71	14.70	-	-	-	-
Unfolded yield	919.9	960.7	1189.6	1184.5	1182.9	1141.5	1027.3	929.2	979.5	979.9	905.8	376.7	6579.1	8535.7	3241.9	11777.6
$\Delta\mathcal{B}$	122.7	117.6	143.6	145.0	153.4	150.2	134.1	121.7	116.0	103.7	86.8	28.7	832.5	1088.3	335.3	1423.5
$\Delta\mathcal{B}$ (Without FSR)	128.0	119.2	144.6	143.7	152.5	149.0	133.3	120.1	115.3	102.3	86.1	28.5	837.1	1090.5	332.3	1422.8
Tracking efficiency	3.1	1.9	3.1	2.3	2.3	3.9	2.6	4.1	3.5	1.3	4.1	9.4	2.3	2.5	2.9	2.6
Photon efficiency	5.8	3.3	2.6	1.3	2.2	2.5	3.1	3.0	5.0	1.4	5.2	24.4	1.9	2.2	4.7	2.7
$K_L^0$ efficiency	0.8	0.3	0.6	0.3	0.5	0.4	0.5	0.8	0.6	0.5	1.7	6.9	0.3	0.3	1.0	0.4
$K_L^0$ production spectrum	0.9	0.6	1.0	0.6	1.1	1.0	0.6	2.8	1.7	1.0	2.0	8.3	0.7	0.9	1.9	1.1
$K_L^0$ energy	1.0	0.6	0.3	0.2	0.2	0.3	0.2	0.4	0.6	0.7	0.8	7.1	0.2	0.3	0.8	0.3
$\ell$ identification	3.8	1.0	1.2	1.3	0.6	0.6	1.6	1.0	0.9	1.6	0.7	4.9	0.3	0.5	1.1	0.6
$\pi$ identification	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	5.6	0.2	0.2	0.7	0.3
Bremsstrahlung	0.5	0.3	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	5.3	0.2	0.2	0.7	0.3
$q^2$ continuum shape	7.6	1.6	0.9	0.3	0.7	0.3	1.1	0.3	0.7	1.2	1.2	5.5	0.9	0.8	1.0	0.7
$m_{ES}$ continuum shape	8.8	0.6	1.1	0.6	0.1	0.5	0.7	0.5	1.1	0.8	1.6	28.3	1.8	1.5	3.4	2.0
$\Delta E$ continuum shape	3.4	2.7	0.4	0.5	0.5	0.1	0.2	0.3	0.3	1.4	2.1	9.0	1.2	0.9	1.8	1.1
$\mathcal{B}(B^+ \rightarrow \pi^0 \ell^+ \nu)$	0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.3	0.4	7.2	0.2	0.2	0.8	0.3
$\mathcal{B}(B^0 \rightarrow \rho^- \ell^+ \nu)$	0.5	0.3	0.1	0.1	0.2	0.1	0.2	0.3	0.3	0.4	0.5	10.2	0.2	0.2	0.9	0.3
$\mathcal{B}(B^+ \rightarrow \rho^0 \ell^+ \nu)$	0.6	0.2	0.1	0.1	0.2	0.2	0.2	0.3	0.4	0.3	0.3	7.4	0.2	0.2	0.9	0.3
$\mathcal{B}(B^+ \rightarrow \omega \ell^+ \nu)$	0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.4	0.3	8.3	0.2	0.2	1.0	0.3
$\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu)$	0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.3	0.2	5.6	0.2	0.2	0.7	0.3
$\mathcal{B}(B^+ \rightarrow \eta' \ell^+ \nu)$	0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.3	0.3	5.4	0.2	0.2	0.7	0.3
Non resonant $b \rightarrow u \ell \nu$ BF	0.6	0.3	0.1	0.1	0.2	0.2	0.2	0.4	0.5	0.3	0.6	7.9	0.2	0.2	0.7	0.3
SF parameters	0.9	0.5	0.9	0.4	0.3	0.5	0.6	0.3	0.5	2.3	4.1	23.5	0.6	0.4	2.3	0.8
$B \rightarrow \rho \ell \nu$ FF	2.3	1.4	2.0	1.4	1.9	1.6	0.8	0.7	2.4	3.0	1.1	16.7	1.8	1.5	1.8	1.5
$B^0 \rightarrow \pi^- \ell^+ \nu$ FF	0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.3	0.3	7.5	0.2	0.2	0.9	0.3
Other scalar FF	1.0	0.3	0.5	0.5	0.4	0.3	0.3	0.7	1.7	2.1	2.1	8.7	0.4	0.3	0.6	0.3
$B \rightarrow \omega \ell \nu$ FF	0.5	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.5	0.6	0.6	18.2	0.2	0.2	1.8	0.5
$\mathcal{B}(B \rightarrow D \ell \nu)$	0.5	0.6	0.2	0.1	0.3	0.2	0.5	0.3	0.4	0.4	0.4	5.7	0.2	0.3	0.7	0.4
$\mathcal{B}(B \rightarrow D^* \ell \nu)$	0.6	0.3	0.2	0.3	0.4	0.3	0.3	0.5	0.4	0.3	0.5	5.7	0.3	0.3	0.7	0.4
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)$	0.7	0.3	0.3	0.5	0.5	0.3	0.8	0.6	1.1	0.7	0.7	5.9	0.3	0.3	0.9	0.4
Non resonant $b \rightarrow c \ell \nu$ BF	0.6	0.2	0.2	0.1	0.2	0.2	0.2	0.5	0.3	0.3	0.3	5.6	0.2	0.2	0.7	0.3
$B \rightarrow D \ell \nu$ FF	0.5	0.2	0.3	0.1	0.2	0.1	0.2	0.4	0.4	0.3	0.4	5.7	0.2	0.2	0.7	0.3
$B \rightarrow D^* \ell \nu$ FF	0.6	0.2	0.2	0.4	0.2	1.0	0.6	2.0	0.4	0.7	1.2	7.0	0.2	0.3	1.0	0.5
$\Upsilon(4S) \rightarrow B^0 \bar{B}^0$ BF	1.4	1.7	1.2	1.3	1.3	1.3	1.5	1.2	1.4	1.4	1.0	5.9	1.3	1.4	1.2	1.3
Secondary lepton	4.1	3.1	2.1	1.2	1.7	0.5	1.2	0.5	0.5	0.9	3.7	5.9	1.0	0.9	1.2	0.9
Final state radiation	0.3	1.3	0.8	2.2	0.3	1.4	1.2	1.3	1.4	1.6	0.8	3.4	1.0	1.1	1.5	1.2
B counting	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Fit bias	0.1	0.3	0.4	0.1	0.1	0.4	0.4	0.7	0.1	1.0	2.0	30.8	0.2	0.0	1.8	0.4
Signal MC stat error	1.3	1.5	1.3	1.6	1.4	1.5	1.4	1.4	1.3	1.5	1.2	2.5	0.6	0.4	0.6	0.3
Total systematic error	15.5	6.9	6.0	5.0	5.0	5.9	5.7	7.1	7.9	6.6	10.4	68.8	5.0	4.9	9.2	5.7
Fit error	14.7	11.9	9.0	9.3	9.4	9.4	10.8	12.5	12.8	12.8	17.6	56.7	3.9	3.7	7.6	3.5
Total error	21.4	13.8	10.8	10.6	10.7	11.1	12.2	14.3	15.0	14.4	20.5	89.2	6.3	6.2	12.0	6.7

TABLE V:  $B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$  yields, efficiencies(%),  $\Delta\mathcal{B}$  ( $10^{-7}$ ) and their relative uncertainties (%).

Decay mode $q^2$ bins (GeV <sup>2</sup> )	$\eta' \ell^+ \nu$	$\eta \ell^+ \nu$ ( $3\pi$ )	$\eta \ell^+ \nu$ ( $\gamma\gamma$ )				$\eta \ell^+ \nu$ ( $3\pi$ and $\gamma\gamma$ combined)			
	Total	Total	0-4	4-8	8-16	Total	0-4	4-8	8-16	Total
Fitted yield	141.0	244.8	279.9	216.8	146.7	643.4	303.9	331.5	252.5	887.9
Systematic error	14.6	14.3	5.1	5.8	34.9	9.6	5.8	6.6	28.1	10.3
Fit error	32.8	25.6	13.9	17.2	33.9	12.0	14.1	14.2	26.6	11.0
Efficiency	0.61	0.59	2.01	2.55	1.42	-	2.53	3.41	1.94	-
Unfolded yield	141.0	244.8	299.1	210.9	133.3	643.4	319.3	334.8	233.9	887.9
$\Delta\mathcal{B}$	242.5	431.5	155.3	86.3	97.7	339.3	131.8	102.6	126.2	360.6
Tracking efficiency	5.2	4.1	3.2	2.4	14.6	2.6	2.1	2.0	11.1	2.8
Photon efficiency	5.6	3.1	10.1	4.3	27.4	7.0	8.0	3.8	9.0	5.7
$K_L^0$ efficiency	2.5	0.7	8.6	2.9	27.2	3.2	1.0	0.5	2.2	0.6
$K_L^0$ production spectrum	2.7	1.4	4.7	1.5	16.2	2.5	0.8	0.5	2.3	1.0
$K_L^0$ energy	1.1	1.4	0.6	0.5	2.5	0.9	0.6	0.4	2.3	1.0
$\ell$ identification	2.0	1.8	0.1	2.7	3.9	1.8	0.2	1.9	3.4	1.8
$\pi$ identification	0.6	0.5	-	-	-	-	0.1	0.2	0.5	0.3
Bremsstrahlung	0.5	0.2	1.6	2.7	22.2	8.0	0.3	0.7	12.3	4.2
Continuum yield	4.9	1.1	-	-	-	-	-	-	-	-
$q^2$ continuum shape	5.2	2.6	2.6	1.5	4.5	0.5	2.4	0.7	2.8	0.3
$\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu)$	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0
$\mathcal{B}(B^+ \rightarrow \pi^0 \ell^+ \nu)$	0.2	0.0	0.4	0.9	5.2	1.9	0.3	0.6	2.9	1.3
$\mathcal{B}(B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu)$	0.4	0.4	0.0	0.1	0.8	0.2	0.1	0.1	1.0	0.4
$\mathcal{B}(B^0 \rightarrow \rho^- \ell^+ \nu)$	0.3	0.5	0.1	1.1	6.9	2.3	0.1	0.6	4.2	1.7
$\mathcal{B}(B^+ \rightarrow \rho^0 \ell^+ \nu)$	0.0	0.3	0.1	0.1	0.5	0.1	0.0	0.1	0.8	0.2
$\mathcal{B}(B^+ \rightarrow \omega \ell^+ \nu)$	0.8	1.1	0.1	0.2	2.6	0.8	0.1	0.1	2.6	0.9
Non resonant $b \rightarrow u \ell \nu$ BF	2.3	3.5	0.4	0.9	9.5	3.1	0.5	0.6	8.6	3.4
$\eta$ BF	3.1	1.2	0.5	0.7	0.7	0.6	0.5	0.6	0.7	0.5
SF parameters	4.3	6.3	1.4	2.7	16.8	6.1	1.5	2.5	14.3	6.2
$B \rightarrow \rho \ell \nu$ FF	0.1	0.7	0.1	2.3	1.7	0.9	0.1	1.5	0.9	0.5
$B^+ \rightarrow \eta^{(\prime)} \ell^+ \nu$ FF	1.1	1.0	0.1	0.1	1.4	0.4	0.1	0.1	1.5	0.6
Other scalar FF	2.9	4.2	7.7	1.4	0.1	3.2	0.7	0.1	0.0	0.2
$B \rightarrow \omega \ell \nu$ FF	1.2	2.1	0.1	0.5	2.8	0.7	0.1	0.4	3.9	1.3
$\mathcal{B}(B \rightarrow D \ell \nu)$	1.6	0.7	0.3	0.7	0.6	0.3	0.3	0.7	0.7	0.4
$\mathcal{B}(B \rightarrow D^* \ell \nu)$	0.3	0.4	0.1	0.8	1.2	0.4	0.1	0.7	1.0	0.4
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)$	2.0	1.2	0.6	0.9	2.5	0.7	0.6	0.7	2.6	0.9
Non resonant $b \rightarrow c \ell \nu$ BF	0.1	0.1	0.2	0.1	0.8	0.2	0.3	0.1	0.4	0.2
$B \rightarrow D \ell \nu$ FF	0.1	0.3	0.1	0.1	0.5	0.2	0.1	0.1	0.7	0.3
$B \rightarrow D^* \ell \nu$ FF	0.6	0.9	0.5	0.9	1.3	0.4	0.5	1.2	1.2	0.4
$\mathcal{B}(\Upsilon(4S) \rightarrow B^0 \bar{B}^0)$	1.1	1.2	1.4	1.1	0.9	1.2	1.4	1.2	1.0	1.2
Secondary lepton	4.2	5.0	1.3	0.7	9.1	2.1	1.2	1.6	9.3	3.0
B counting	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Signal MC stat error	1.2	1.1	1.4	1.6	1.2	0.7	1.3	1.3	1.0	0.5
Total systematic error	14.3	12.4	17.0	8.7	55.4	14.1	9.3	6.6	28.7	11.6
Fit error	32.8	25.6	14.6	21.0	39.3	13.7	15.2	16.6	30.3	12.5
Total error	35.8	28.4	22.4	22.7	67.9	19.6	17.8	17.8	41.8	17.0

TABLE VI: Correlation matrix of the partial  $\Delta\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu, q^2)$  statistical and systematic uncertainties.

$q^2$ bins (GeV <sup>2</sup> )	statistical			systematic		
	0-4	4-8	8-16	0-4	4-8	8-16
0-4	1.00	-0.08	0.00	1.00	0.36	0.05
4-8	-0.08	1.00	-0.06	0.36	1.00	0.29
8-16	0.00	-0.06	1.00	0.05	0.29	1.00

TABLE VII: Correlation matrix of the partial  $\Delta\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu, q^2)$  statistical uncertainties.

$q^2$ bins (GeV <sup>2</sup> )	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-26.4
0-2	1.00	-0.16	0.17	0.02	-0.02	0.03	0.01	0.04	0.05	0.02	0.04	-0.00
2-4	-0.16	1.00	-0.32	0.11	0.00	-0.00	-0.01	0.01	0.01	-0.00	0.00	-0.00
4-6	0.17	-0.32	1.00	-0.30	0.15	0.02	0.06	0.06	0.07	0.00	0.01	0.01
6-8	0.02	0.11	-0.30	1.00	-0.22	0.13	0.07	0.06	0.07	0.00	0.00	0.02
8-10	-0.02	0.00	0.15	-0.22	1.00	-0.22	0.16	0.05	0.08	0.01	-0.00	0.02
10-12	0.03	-0.00	0.02	0.13	-0.22	1.00	-0.15	0.10	0.07	-0.01	0.02	0.00
12-14	0.01	-0.01	0.06	0.07	0.16	-0.15	1.00	-0.16	0.13	-0.01	0.05	-0.00
14-16	0.04	0.01	0.06	0.06	0.05	0.10	-0.16	1.00	-0.01	0.01	-0.02	-0.02
16-18	0.05	0.01	0.07	0.07	0.08	0.07	0.13	-0.01	1.00	-0.17	0.09	-0.08
18-20	0.02	-0.00	0.00	0.00	0.01	-0.01	-0.01	0.01	-0.17	1.00	0.05	-0.05
20-22	0.04	0.00	0.01	0.00	-0.00	0.02	0.05	-0.02	0.09	0.05	1.00	-0.35
22-26.4	-0.00	-0.00	0.01	0.02	0.02	0.00	-0.00	-0.02	-0.08	-0.05	-0.35	1.00

TABLE VIII: Correlation matrix of the partial  $\Delta\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu, q^2)$  systematic uncertainties.

$q^2$ bins (GeV <sup>2</sup> )	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-26.4
0-2	1.00	-0.19	0.41	0.33	0.49	0.42	0.49	0.35	0.39	0.14	0.47	0.56
2-4	-0.19	1.00	-0.17	0.08	-0.20	-0.07	-0.14	-0.16	-0.31	0.41	-0.24	0.05
4-6	0.41	-0.17	1.00	0.78	0.82	0.76	0.67	0.68	0.53	0.33	0.72	0.48
6-8	0.33	0.08	0.78	1.00	0.71	0.74	0.65	0.63	0.47	0.49	0.55	0.38
8-10	0.49	-0.20	0.82	0.71	1.00	0.74	0.70	0.70	0.49	0.36	0.66	0.39
10-12	0.42	-0.07	0.76	0.74	0.74	1.00	0.74	0.80	0.61	0.36	0.61	0.40
12-14	0.49	-0.14	0.67	0.65	0.70	0.74	1.00	0.69	0.73	0.29	0.55	0.36
14-16	0.35	-0.16	0.68	0.63	0.70	0.80	0.69	1.00	0.71	0.35	0.64	0.37
16-18	0.39	-0.31	0.53	0.47	0.49	0.61	0.73	0.71	1.00	-0.01	0.62	0.29
18-20	0.14	0.41	0.33	0.49	0.36	0.36	0.29	0.35	-0.01	1.00	0.04	0.20
20-22	0.47	-0.24	0.72	0.55	0.66	0.61	0.55	0.64	0.62	0.04	1.00	0.52
22-26.4	0.56	0.05	0.48	0.38	0.39	0.40	0.36	0.37	0.29	0.20	0.52	1.00

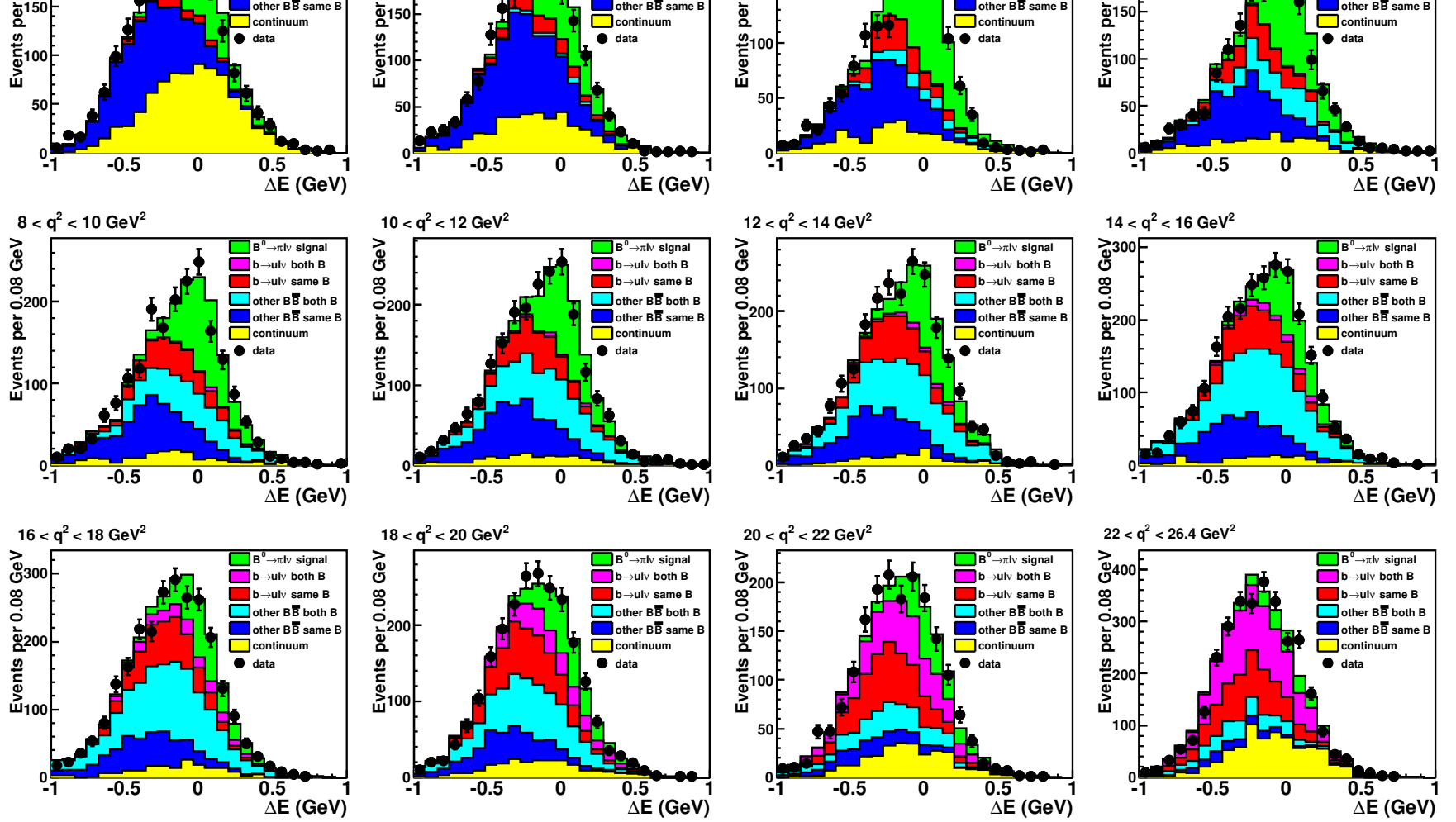


FIG. 5: (color online)  $\Delta E$  yield fit projections in the signal-enhanced region, with  $m_{ES} > 5.2675 \text{ GeV}/c^2$ , obtained in 12  $q^2$  bins from the fit to the experimental data for  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays. The fit was done using the full  $\Delta E$ - $m_{ES}$  fit region.

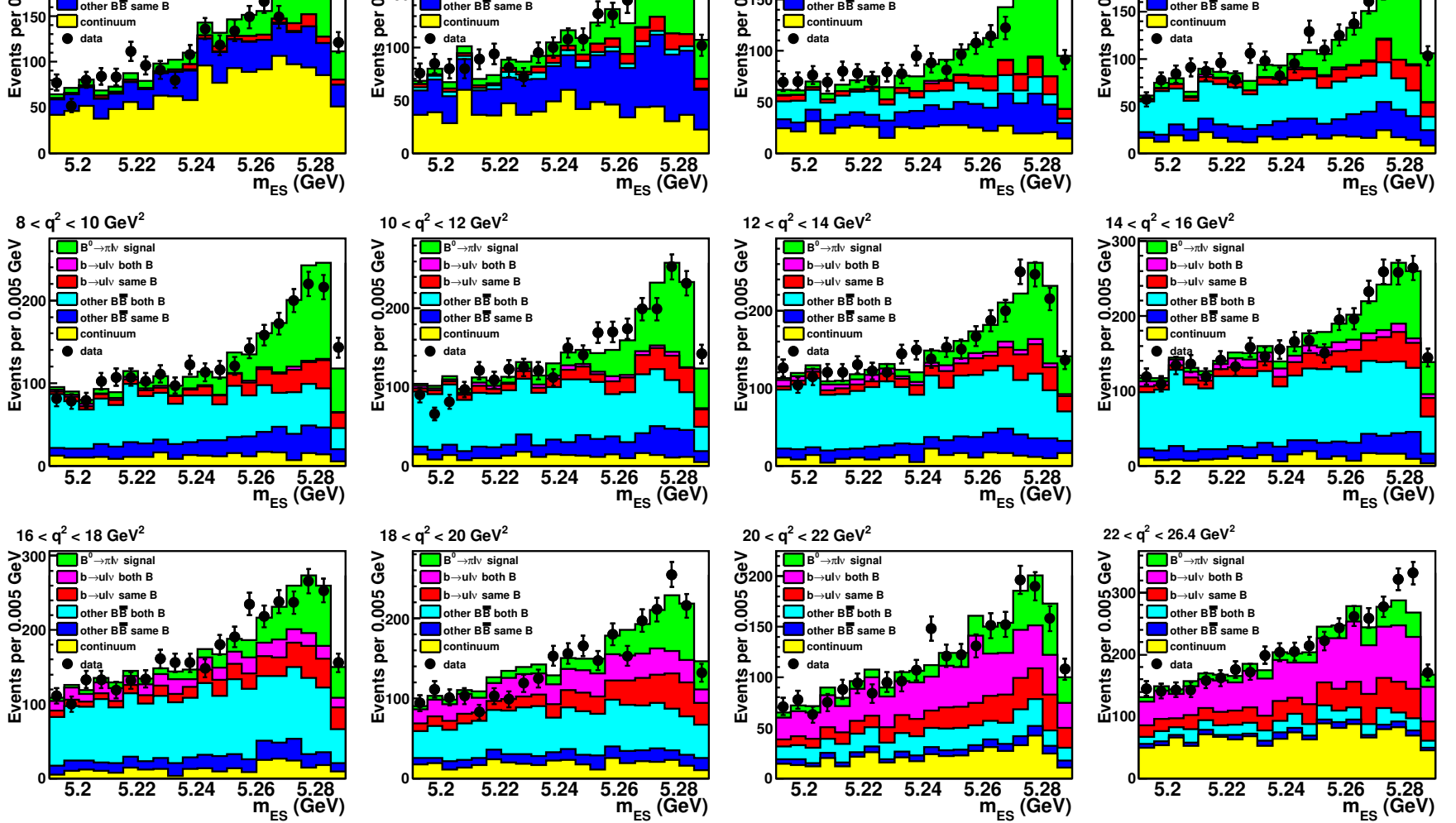


FIG. 6: (color online)  $m_{ES}$  yield fit projections in the signal-enhanced region, with  $-0.16 < \Delta E < 0.2$  GeV, obtained in 12  $q^2$  bins from the fit to the experimental data for  $B^0 \rightarrow \pi^- \ell^+ \nu$  decays. The fit was done using the full  $\Delta E$ - $m_{ES}$  fit region.